# AN ANALYSIS OF FILM FLOW AND ITS APPLICATION TO CONDENSATION IN A HORIZONTAL TUBE

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Abstract—An analysis is presented for turbulent flow in a liquid film which is being dragged along a horizontal tube by an axial shear at the film surface. While being dragged along, the film drains down the wall due to gravity. The analysis can be applied to a number of horizontal, annular, gasliquid flow problems and an example is given here of its use in analysing condensation in a horizontal tube. For this problem, the predictions show limited agreement with the little experimental data presently available.

#### 1. INTRODUCTION

There are a number of important practical problems in which a thin liquid film is being dragged along by surface shear imparted by a gas flow while the film is also being acted upon by gravitational forces in a direction perpendicular to the main flow direction. Examples of these flows are horizontal, annular, two-phase flow, condensation inside a horizontal tube and evaporation inside a horizontal tube.

While studying this type of flow, the author found it necessary to develop a theoretical model for the film flow which takes account of turbulence in the film. This model is presented here and then applied to condensation inside a horizontal tube. Butterworth & Pulling (1974) have also used the model to elucidate the mechanisms occurring in gas-liquid annular flow in a horizontal tube.

The analysis of the film involves a number of assumptions, the main ones of which are as follows:

- (1) The films are sufficiently thin for the usual boundary-layer simplifications to apply.
- (2) The flow in the film is not changing rapidly with axial distance nor with time.
- (3) The axial flow in the film is much greater than the circumferential flow. This, as is shown, allows a great simplification of the equations since the inertial terms may be neglected.
- (4) The turbulent eddy-viscosity concept may be used to represent the momentum transfer in turbulent flow.
- (5) The eddy viscosity is assumed to be isotropic and governed by the axial flow. The latter part of the assumption follows from the assumption that the axial flow is much greater than the circumferential flow.
- (6) The eddy viscosity profile is assumed to be the same as that for single-phase flow in a tube.

## 2. ANALYSIS OF FILM HYDRODYNAMICS

# 2.1 Axial flow in the film

Consider a point on the tube wall at an angular position  $\theta$  from the vertical: this is shown in figure 1. The rectangular coordinate system illustrated in this figure is used in the analysis. In this, x is the tangential direction, y the radial (inward from the wall) direction and z the axial direction. With the assumptions given in the Introduction, the axial equation of motion for the film is

$$u_{x}\frac{\partial u_{z}}{\partial x} + u_{y}\frac{\partial u_{z}}{\partial y} = \frac{\partial}{\partial y}\left\{ (v + \varepsilon)\frac{\partial u_{z}}{\partial y} \right\}$$
[1]

where  $u_x$ ,  $u_y$  and  $u_z$  are the velocities in the x, y and z directions respectively, v is the molecular kinematic viscosity and  $\varepsilon$  the turbulent diffusivity of momentum.

It is shown in the Appendix that the inertial terms in [1] may be neglected provided that

$$\Gamma_x^+ \frac{\delta}{R} \frac{v}{\bar{\epsilon} + v} \ll 1$$
 [2]

where  $\delta$  is the film thickness, R the tube radius and  $\bar{\varepsilon}$  the average of  $\varepsilon$  across the film.  $\Gamma_x^+$  is the dimensionless circumferential film flow:

$$\Gamma_x^+ = \Gamma_x/\mu \tag{3}$$

where  $\Gamma_x$  is the circumferential mass flow per unit length of tube and  $\mu$  is the viscosity.

Neglecting the inertial terms, [1] becomes

$$\frac{\mathrm{d}}{\mathrm{d}y}\left\{(v+\varepsilon)\frac{\mathrm{d}u_z}{\mathrm{d}y}\right\} = 0.$$
[4]



Figure 1. Coordinate system used in the analysis of the film.

For thin films, the shear stress may be assumed to be independent of y. Hence [4] may be integrated to give

$$\left(1 + \frac{\varepsilon}{\nu}\right)\frac{\mathrm{d}u_z}{\mathrm{d}y} = \frac{\tau_o}{\mu} \tag{5}$$

where  $\tau_{o}$  is the wall shear stress.

If we assume that the turbulent diffusivity profile is the same as that for single-phase flow in a tube, [5] can be integrated to give the usual universal velocity profile for turbulent flow in a tube.

$$u_z^+ = y^+, y^+ \le 5$$
 [6a]

$$u_z^+ = -3.05 + 5 \ln y^+, 5 < y^+ \le 30$$
 [6b]

$$u_z^+ = 5.5 + 2.5 \ln y^+, y^+ > 30$$
 [6c]

where  $u_z^+ = u_z/u^*$  and  $y^+ = u^* y/v$ , where  $u^* = \sqrt{(\tau_o/\rho)}$ .

Using [5], it is evident that [6] is consistent with the following turbulent diffusivity profile

$$\frac{\varepsilon}{v} = 0, y^+ \le 5$$
 [7a]

$$\frac{\varepsilon}{v} = \frac{y^+}{5} - 1, 5 < y^+ \le 30$$
 [7b]

$$\frac{\varepsilon}{v} = \frac{y^+}{2.5} - 1, y^+ > 30.$$
 [7c]

The dimensionless axial film flow  $\Gamma_z^+$  (which is  $\Gamma_z/\mu$  where  $\Gamma_z$  is the axial mass flow in the film per unit perimeter of tube) is calculated by integrating the velocity profile across the film:

$$\Gamma_{z}^{+} = \int_{0}^{\delta^{+}} u_{z}^{+} \, \mathrm{d}y^{+}.$$
 [8]

Using [6] and [8] gives

$$\Gamma_z^+ = \frac{1}{2} (\delta^+)^2, \, \delta^+ \le 5 \tag{9a}$$

$$\Gamma_z^+ = 12.5 - 8.05\,\delta^+ + 5\,\delta^+ \ln\delta^+, \, 5 < \delta^+ \le 30$$
[9b]

$$\Gamma_z^+ = -64 + 3\,\delta^+ + 2.5\,\delta^+ \ln\delta^+, \,\delta^+ > 30.$$
[9c]

There is nothing novel about the results of the film flow analysis which have been presented so far. Similar results have been obtained, with slight variations, by Anderson & Mantzouranis (1960), Travis *et al.* (1971) and many others. The difference here comes in showing that the analysis applies with circumferential flow superimposed on the main axial flow. Further, it is clear that there are now more advanced methods of representing the turbulence in the film although these methods do not lead to a significant improvement in accuracy in this type of problem (Hewitt & Hall Taylor 1970). The present approach has the useful feature that we can quickly obtain solutions without recourse to lengthy analyses which may involve numerical integration. The reason for keeping to a simple turbulence model will become clearer in the subsequent sections where the analysis becomes more difficult.

# 2.2 Circumferential flow in the film

The circumferential velocity in the film is represented by

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \frac{\partial}{\partial y} \left\{ (v + \varepsilon) \frac{\partial u}{\partial y} \right\} + (1 - \rho_G/\rho) g \sin\theta$$
[10]

where  $\rho_{G}$  and  $\rho$  are the gas and liquid phase viscosities, respectively and g is the gravitational acceleration.

As before, we can neglect the inertial terms on the left-hand side of [10] provided that [2] is satisfied. Integrating [10] therefore gives

$$(v + \varepsilon) \frac{\mathrm{d}u_x}{\mathrm{d}y} = -y \left(1 - \frac{\rho_{\mathrm{G}}}{\rho}\right) g \sin \theta + c.$$
 [11]

The constant of integration c is determined by  $du_x/dy = 0$  when  $y = \delta$ . Hence [11] becomes

$$\frac{\mathrm{d}u_x}{\mathrm{d}y} = \frac{(\delta - y)(1 - \rho_{\mathrm{G}}/\rho)\,\mathrm{g\,\sin\theta}}{v + \varepsilon}.$$
[12]

This equation may be rewritten in dimensionless form as

$$\frac{\mathrm{d}u_x^+}{\mathrm{d}y} = B \frac{\delta^+ - y^+}{1 + \frac{\varepsilon}{v}}$$
[13]

where

$$B = \frac{g v (1 - \rho_{\rm G}^{-}/\rho) \sin \theta}{(u^*)^3}$$
[14]

and  $\delta^+ = \delta u^* / v$  and  $u_x^+ = u_x / u^*$ .

If we assume that the turbulence is isotropic, we can use the turbulent diffusivity profile from [7] in [13] to give, on integration,

$$u_x^+ = B\{\delta^+ y^+ - \frac{1}{2}(y^+)^2\}, y^+ \le 5$$
 [15a]

$$u_x^+ = B\left\{\delta^+ - y^+ + \delta^+ \ln \frac{y^+}{5} + \frac{5}{2}\right\}, 5 < y^+ \le 30$$
 [15b]

$$u_x^+ = B\left\{\delta^+(1+\ln 6) - \frac{y^+}{2}\ln\frac{\delta^+}{30} - \frac{25}{2}\right\}, y^+ > 30.$$
 [15c]

The circumferential dimensionless film flow  $\Gamma_x^+$  is calculated from

$$\Gamma_x^+ = \int_0^{\delta^+} u_x^+ dy^+$$
 [16]

by substituting for  $u_x^+$  from [15]:

$$\Gamma_x^+ = B\phi(\delta^+)\sin\theta \qquad [17]$$

where

$$\beta = gv(1 - \rho_G/\rho)/(u^*)^3$$
[18]

i.e.

 $B=\beta\sin\theta$ 

where the function  $\phi$  is given by

$$\phi = \frac{1}{3}(\delta^+)^3, \, \delta^+ \le 5 \tag{[19a]}$$

$$\phi = 5\left\{5\delta^+ + (\delta^+)^2 \left(\ln\frac{\delta^+}{5} - \frac{1}{2}\right) - \frac{25}{6}\right\}, 5 < \delta^+ \le 30$$
[19b]

$$\phi = 5\left\{\frac{1325}{6} - 25\delta^{+} + (\delta^{+})^{2} \left(\frac{1}{4} + \ln 6 + \frac{1}{2}\ln\frac{\delta^{+}}{30}\right)\right\}, \delta^{+} > 30.$$
 [19c]

Gardner (1972) has pointed out that circumferential velocities in the film cannot be greater than the free fall velocity. This gives therefore a further restriction on the validity of the present analysis which must be checked for whenever the analysis is used.



Figure 2. Illustration of Chaddock-Chato model for intube condensation.

675

#### 3. APPLICATION TO CONDENSATION IN A HORIZONTAL TUBE

## 3.1 Background

A number of analyses of condensation in horizontal tubes (e.g. Chato 1962; Chaddock 1957) make the assumption illustrated in figure 2 that the tube may be divided into two regions around the circumference. In the upper portion of the tube, the condensate flow is assumed to be entirely circumferential and, in the lower portion, the accumulated condensate flows axially to the tube exit. The upper-region, heat transfer coefficients and condensate behaviour are determined by the Nusselt (1916) type of analysis which was applied initially to condensation outside a tube. It is evident, however, that the Nusselt-type analysis is a simplification of the actual behaviour since, in practice, the condensate film in the upper portion of the tube undergoes some axial flow due to drag from the uncondensed vapour. The methods used above in this paper are therefore applied here to analyse more fully the upper portion of the tube.

### 3.2 Assumptions

In general, the assumptions are those made previously but, for clarity, these are restated here in the context of condensation. It is assumed firstly that [17] is valid. This, of course, implies that condition [2] is satisfied. This is ensured here as the analysis is restricted to the top of the tube. We further assume that the rate of change of conditions with axial distance is small. Thus, the analysis does not apply to the inlet region of the tube where there is an initial buildup of axial film flow. For convenience, we ignore circumferential variations in wall shear stress. For intube condensation, such variations occur because of the changes in condensation rate around the tube and because the gas phase is effectively flowing in a tube with roughness changes around the perimeter caused by asymmetric waves on the film. Finally, the entrainment and deposition fluxes are ignored in comparison with the film drainage rates and condensation fluxes.

#### 3.3 Heat transfer in the condensate film

At a given depth in the film, the heat flux q is related to the temperature gradient as follows

$$q = \rho C(\alpha + \varepsilon) \frac{\mathrm{d}I}{\mathrm{d}y}$$
[20]

where C is the liquid specific heat,  $\alpha$  the molecular diffusivity of heat,  $\varepsilon$  the turbulent diffusivity of heat and T the temperature. We can define a heat-transfer coefficient h for condensation as follows

$$h = q/(T_{\delta} - T_0)$$
<sup>[21]</sup>

where  $T_{\delta}$  and  $T_0$  are the temperatures at  $y = \delta$  and y = 0 respectively (i.e. the film surface and tube wall).

Equation [20] may be integrated and rearranged using [21] to give

$$h = \rho C u^* / T^+$$
 [22]

where

$$T^{+} = \int_{0}^{\delta^{+}} \frac{\mathrm{d}y^{+}}{1/\mathrm{Pr} + \varepsilon/\nu}$$
[23]

where Pr is the condensate Prandtl number.

Now, we can make the usual assumption that the turbulent diffusivity of heat is the same as that of momentum. Combining [23] and [7] therefore gives

$$T^+ = \delta^+ \operatorname{Pr}, \, \delta^+ \le 5$$
[24a]

$$T^{+} = 5 \left[ \Pr + \ln \left\{ 1 + \Pr \left( \frac{\delta^{+}}{5} - 1 \right) \right\} \right], 5 < \delta^{+} \le \Pr$$
[24b]

$$T^{+} = 5 \left[ \Pr + \ln(1 + 5\Pr) + \frac{1}{2} \ln \frac{\delta^{+}}{30} \right], \delta^{+} > 30.$$
 [24c]

The last term in [24c] strictly only follows from [23] and [7] when Pr = 1. It is, however, a good approximation in other circumstances. Equations very similar, or identical, to these have been derived many times before (e.g. Travis *et al.* 1971) for one-dimensional flow in the film.

# 3.4 Continuity equation

Using the assumptions in Section 3.2 above, the continuity equation is

$$\frac{\mathrm{d}\Gamma_x}{\mathrm{d}\theta} = Rm_c \tag{25}$$

where  $m_c$  is the condensation mass flux which is related to the heat transfer coefficient as follows

$$h = \frac{m_c \lambda}{\Delta T}$$
[26]

where  $\lambda$  is the latent heat and  $\Delta T$  is  $T_{\delta} - T_0$ .

Combining [25] and [26] and writing in terms of the previous dimensionless groups gives

$$\frac{\mathrm{d}\Gamma_x^+}{\mathrm{d}\theta} = \frac{R\Delta T u^* k}{v^2 \lambda \rho} \frac{\mathrm{Pr}}{T^+}$$
[27]

where k is the condensate thermal conductivity.

Combining [17] and [27] gives

$$\frac{\mathrm{d}\delta^{+}}{\mathrm{d}\theta} = \frac{\frac{1}{2}A\operatorname{Pr}/T^{+} - \phi\cos\theta}{\frac{\mathrm{d}\phi}{\mathrm{d}\delta^{+}}\sin\theta}$$
[28]

677

where

$$A = \frac{D\Delta T k(u^*)^4}{v^3 \lambda g(\rho - \rho_G)}.$$
[29]

At the top of the tube, the denominator of the r.h.s. of [28] is zero. Since  $d\delta^+/d\theta$  must be finite, the numerator must also be zero. Hence, the film thickness at the top of the tube may be determined, by solving the following equation for  $\delta^+$ 

$$A = 2T^+ \phi/\Pr.$$
 [30]

Having determined  $\delta^+$  at the top of the tube, [28] could be integrated around the perimeter to give the full distribution of  $\delta^+$  and hence, from [22] and [24] the full variation of heat-transfer coefficient around the tube.

# 3.5 Comparison with the Nusselt solution at the top of the tube

If we solve [30] for  $\delta^+ \leq 5$ , we obtain  $\delta^+$  at the top of the tube as follows

$$\delta^+ = \left(\frac{3}{2}A\right)^{1/4}.$$
[31]

Substituting this back into [24a] and [22] gives

$$h = \frac{\rho C u^*}{\Pr(3A/2)^{1/4}}.$$
 [32]

Substituting for A from [29] gives

$$h = \left[\frac{2k^3\lambda(\rho - \rho_G)g}{3D\Delta T\nu}\right]^{1/4}$$
[33]

which is the Nusselt solution for *local* coefficients at the top of the tube. Note that this solution is obtained for all  $\delta^+$  provided that there is no turbulence in the film. In other words, the Nusselt solution still applies even with a superimposed axial film flow provided that the axial film flow is insufficient to induce turbulence.

It is of interest to compare the coefficients obtained with a turbulent film with those obtained with the Nusselt laminar film solution for the same conditions. Using [22] and [32] gives

$$\frac{h}{h_{\rm Nu}} = \left(\frac{3}{2}A\right)^{1/4} \frac{\Pr}{T^+}$$
[34]

where  $h_{Nu}$  is the Nusselt-solution coefficient obtained from [32]. By varying  $\delta^+$ , it is possible to obtain  $h/h_{Nu}$  as a function of the independent variable A. Figure 3 shows a plot of  $h/h_{Nu}$  against A for different Prandtl numbers.

Unfortunately, there is no systematic data to test out this theory. A preliminary examination of the results obtained at Harwell on the in-tube condenser rig has suggested that [34] predicts too low. Some experimental points are indicated in figure 3. These data are for

678



Figure 3. Effect of shear stress on condensation heat transfer coefficients at the top of the tube.

condensing *n*-propyl alcohol in a 24.4 mm i.d. tube. Further details of the equipment and the experimental technique are given by Butterworth, Hazell & Pulling (1974).

The wall shear stress for use in the method was calculated from

$$\tau_0 = \frac{R}{2} \left( -\frac{\mathrm{d}p_{\mathrm{F}}}{\mathrm{d}z} \right)$$
[35]

where  $dp_F/dz$  is the frictional component of the pressure gradient which was calculated from the accurate proprietary correlation developed by the Heat Transfer and Fluid Flow Service, Harwell.

One explanation for the underprediction is that waves on the film surface increase the mixing within the film and, therefore, increase the heat transfer coefficient. Shekriladze and Mestvishvili (1973) have suggested that this effect should introduce a correction factor of 1.7 for condensation with vapour shear. Such a correction would be more than enough to account for the discrepancy shown in figure 3.

# 4. CONCLUSIONS

The following main conclusions follow from this paper:

(1) For the film flow problem considered a criterion is given for when the inertial terms in the Navier–Stokes equations can be neglected.

(2) By ignoring the inertial terms, and introducing a turbulent diffusivity profile induced by the axial flow, axial and circumferential velocity profiles for the film are derived. From these, the axial and circumferential flows are calculated. (3) The new model is applied to the problem of condensation inside a horizontal tube. It was shown that, when turbulence is induced by the axial flow, the heat transfer coefficients are increased above those predicted by the Nusselt analysis. The enhancements predicted, however, appear to be less than those observed experimentally. This discrepancy may be due to waves.

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#### APPENDIX

Order of magnitude analysis to show when the inertial terms may be neglected

This analysis is applied for the simpler case of laminar flow to illustrate the method. The turbulent flow analysis is similar and the required result for turbulent flow is quoted.

For fully-developed flow and thin films, the Navier-Stokes equation of motion for axial laminar flow is

$$u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} = \frac{\partial^2 u_z}{\partial y^2}.$$
 [A1]

Introducing the following dimensionless groups

$$U_x = \frac{u_x}{\bar{u}_z}, U_y = \frac{u_y}{\bar{u}_z} \text{ and } U_z = \frac{u_z}{\bar{u}_z}$$
 [A2]

where  $\bar{u}_z$  is the mean axial film velocity (at position  $\theta$ ),  $x = x/\delta$  and  $Y = y/\delta$ , where  $\delta$  is the film thickness.

Equation [A1] therefore becomes

$$\operatorname{Re}_{z}\left(U_{x}\frac{\partial U_{z}}{\partial X}+U_{y}\frac{\partial U_{z}}{\partial Y}\right)=\frac{\partial^{2}U_{z}}{\partial Y^{2}}$$

$$\operatorname{Re}_{z}\frac{\tilde{u}_{x}}{\tilde{u}_{z}}\cdot\frac{1}{\dot{R}/\delta}\qquad\operatorname{Re}_{z}\frac{\bar{u}_{y}}{\bar{u}_{z}}\cdot\frac{1}{1}\qquad\frac{1}{1}.$$
[A3]

Following the order of magnitude analysis of Schlichting (1968), I have written the orders of magnitude below each term in this equation. In [A3], Re<sub>z</sub> is the Reynolds number for axial flow in the film (i.e.  $\bar{u}_z \delta/\nu$ ).

The continuity equation for the film is

$$\frac{\partial U_x}{\partial X} + \frac{\partial U_y}{\partial Y} = 0$$

$$\frac{\bar{u}_x / \bar{u}_z}{R / \delta} \quad \frac{\bar{u}_y / \bar{u}_z}{1}$$
[A4]

since  $\partial u_z/\partial z = 0$  for fully-developed flow. Again, the orders of magnitude are written below each term. It follows from [A4] that

$$\bar{u}_y = 0\left(\frac{\delta}{R}\,\bar{u}_x\right).$$

Hence, both terms on l.h.s. of [A3] are of order  $\operatorname{Re}_z(\bar{u}_x/\bar{u}_z)(\delta/R)$ . If we assume that this group is very much less than unity, [A3] may be written as

$$0 = \frac{\mathrm{d}^2 U_z}{\mathrm{d}Y^2}.$$
 [A5]

As yet, we do not know whether  $\operatorname{Re}_{z}(\bar{u}_{x}/\bar{u}_{z})/(\delta/R)$  is small, but this can be checked for in the specific problem being analysed.

In the analysis of a similar problem, that of laminar film flow down an inclined rod, Butterworth (1967) assumed, without proof, that the inertial terms in the Navier-Stokes equation may be neglected. The resultant predictions were in good agreement with the experimental data. The above argument explains why that analysis was so successful.

For turbulent film flow, the analysis is similar but starts with [1]. The result is that

$$\operatorname{Re}_{z} \frac{\bar{u}_{x}}{\bar{u}_{z}} \frac{\delta}{R} \frac{v}{\bar{v} + v} \ll 1$$
[A6]

for the inertial terms to be neglected, where  $\bar{\epsilon}$  is the average of  $\epsilon$  across the film. From the magnitude of the terms in [A4], [A6] may be rewritten as

$$\operatorname{Re}_{x}\frac{\delta}{R}\frac{v}{\bar{\varepsilon}+v}\ll1$$
[A7]

which becomes [2] in the text since  $\operatorname{Re}_{r}$  is the same as  $4\Gamma_{r}^{+}$ .

**Résumé**—On présente une analyse de l'écoulement turbulent ou laminaire dans un film liquide qui est entrainé dans une conduite horizontale par un effort de cisaillement axial appliqué à la surface du film. Tout en étant entrainé, le film présent sur la partie supérieure de la paroi s' appauvrit par effet de gravité. L'analyse est appliquée à des écoulements adiabatiques complètement developpés d'une part dans l'hypothèse d'une alimentation du Film par déposition de gouttes, et d'autre part dans l'hypothèse d'une alimentation par condensation à l'intérieur de la conduite. Les résultats ne sont compatibles dans aucun des cas avec les données expérimentales mais les écarts peuvent etre du à des facteurs qui ne sont pas inclus dans le modèle du film liquide.

Auszug—Es wird eine Analyse der turbulenten Stroemung in einer Fluessigkeitsschicht angegeben, die durch axialen Schub in der Schichtoberflaeche laengs eines wagerechten Rohres verschoben wird. Waehrend dieser Laengsverschiebung fliesst die Schicht unter Schwerkraftseinfluss auf der Wand nach unten ab. Die Analyse wird zunaechst auf voll entwickelte adiabatische Stroemungen angewandt, wobei vorausgesetzt wird, dass die abfliessende Menge durch Tropfenanlagerung ersetzt wird, dann auf Kondensation im Rohr, wobei Schichtauffuellung durch Kondensation angenommen wird. In keinem der beiden Faelle stimmen die Resultate mit Versuchsergebnissen ueberein, doch sind die Unterschiede moeglicherweise auf Faktoren zurueckzufuehren, die ausserhalb des Modells fuer die Fluessigkeitsschicht liegen.

Резюме—Представлен анализ ламинарного и турбулентного течений в жидких пленках, которые протягиваются вдоль горизонтальной трубы под действием осевого сдвига по поверхности пленки. Будучи протягиваемы, эти пленки стекают вниз по трубе под влиянием силы тяжести. Первоначально анализ приложен к хорошо развитыму адиабатическим потокам в предположеним, что поток пополняется за счет капельных выпадений, а позднее в предположении пополнения за счет внутритрубной конденсации. Ни в одном из указанных случаев полученные результаты не были сходны с экспериментальными данными, но это несоответствие может быть отнесено факторов, находящихся вне данной модели жидкой пленки.